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A Novel Three-Parameter Distribution for lifetime data with application to COVID-19 second wave dataset in Nepal

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ABSTRACT
In this present study, we have introduced a novel three-parameter distribution (NTPD) designed for modeling lifetime data, showcasing its remarkable performance in reliability and survival analysis. We have presented expressions for a range of statistical functions, including the probability density function, distribution function, survival function, quantile function, hazard rate function, reversed hazard rate function, cumulative hazard rate function, skewness, and kurtosis. Visual representations of the probability density and hazard rate curves have also been provided. To assess the suitability and effectiveness of our proposed model, we employed a COVID-19 second wave dataset from Nepal. We estimated the model parameters using three different techniques: maximum likelihood, least squares, and Cramer's-von Mises. To confirm the model's validity, we employed a range of statistical criteria, such as Akaike's Information Criterion, Bayesian Information Criterion, Corrected Akaike's Information Criterion, and Hannan-Quinn Information Criterion. Additionally, P-P and Q-Q plots were used for validation purposes. To assess how well the data fits, we performed the Kolmogorov-Smirnov, Anderson-Darling, and Cramer-von Mises tests. These tests were carried out to determine the adequacy of the fit for our data. Our empirical findings demonstrate that, when compared to alternative lifetime distributions, the suggested distribution exhibits superior fitting and greater flexibility for lifetime data analysis. The utilization of the R programming language facilitated robust and insightful data analysis, leading to valuable insights.

KEYWORDS
Bayesian Information Criterion, COVID-19, Goodness of fit, Maximum Likelihood Estimation, novel three-parameter distribution, second wave, survival function.

1. Introduction

Lifetime distribution, also known as survival distribution or failure time distribution, plays a pivotal role in various fields such as reliability engineering, survival analysis, biology, medicine, finance, economics, social sciences, and actuarial science. It models the probability distribution of the time until an event of interest occurs, whether it is the failure of a mechanical component, the survival time of patients, or the lifespan of a product. The analysis of lifetime distributions provides valuable insights into

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the reliability and durability of systems, allowing for informed decision-making and risk assessment. Understanding the characteristics and parameters of lifetime distributions is essential for predicting and managing events that occur over time. Studying continuous probability distributions, such as the exponential, Cauchy, and Weibull distributions, is a common practice in statistical literature for the analysis of lifetime data. These probability distributions play a crucial role in understanding and modeling the variability in lifetimes, making them essential tools in fields like reliability engineering, survival analysis, and actuarial science. By examining these distributions, researchers and analysts can gain valuable insights into the behavior of data points over time, allowing them to make informed decisions and predictions in various real-world scenarios. In recent years, considerable attention from researchers has been directed towards the exponential distribution due to its efficacy in modelling lifetime data. Its favourable attributes stem from closed-form solutions available for numerous survival analyses. However, while the exponential distribution is often justifiable under the assumption of a constant failure rate, real-world failure rates tend to exhibit variability. Consequently, relying on the exponential lifetime model in a random manner can be both inadequate and unrealistic. More recently, novel classes of models have emerged by building upon modifications of established classical probability models, as demonstrated by (Marshall & Olkin ,2007). Contemporary efforts have been dedicated to devising fresh distributions that expand upon existing ones, thereby enhancing the flexibility of data modelling practices. Through the inclusion of supplementary criteria, various techniques can be employed to extend the scope of existing established distributions, resulting in the creation of broader families of models. This trend has given rise to multiple categories within statistical literature, introducing one or more parameters to generate innovative models, as illustrated by the works of (Pham & Lai ,2007)and (Rinne,2009).Some of the well-known life time models found in the literature are Weibull distribution (Weibull, 1951), Lindley distribution (Lindley, 1958), Inverse Weibull (Keller et al. ,1982), Exponentiated Weibull (Mudholkar et al., 1995), Exponential power (Srivastava & Kumar, 2011), A new two-parameter lifetime distribution (Alizadeh et al. ,2019), Logistic-exponential power (Joshi et al.,2020), A Two Parameter New Distribution (Chaudhary & Kumar,2020), A new three parameter lifetime model(Muhammad et al.,2021), Inverse exponentiated odd Lomax exponential distribution(Chaudhary et al.,2022),Modified Upside Down Bathtub-Shaped Hazard Function Distribution(Chaudhary et al.,2023),and the inverse exponential power distribution (Chaudhary et al., 2023). This paper presents a new class of life time distributions called a novel three-parameter distribution (NTPD) for lifetime data, demonstrating its superior performance in reliability/survival analysis. The article's primary goal is to propose a more adaptable model that achieves improved fitting accuracy for lifetime datasets. The following structure is used to present the various sections of this study. In Section 2, we will introduce the novel three-parameter distribution (NTPD) while elucidating its mathematical and statistical properties. Moving on to Section 3, we will delve deeply into the estimation techniques, which will include discussions on least-squares (LSE), Cramer-Von-Mises (CVME), and maximum likelihood (MLE). In Section 4, our focus will be on providing model parameter estimates, utilizing data from the COVID-19 second wave in Nepal. Furthermore, we will present examples of the different criteria employed to assess the goodness of fit of the proposed model. In concluding section 5, this study has strived to contribute valuable insights to the field of statistical analysis and modeling. We hope that the information presented in this paper serves as a valuable resource for researchers, practitioners, and policymakers alike.

2. Model Analysis

In this research, we have created a new life time distributions called a novel three-parameter distribution (NTPD) for lifetime data. The Cumulative distribution function(cdf) of NTPD distribution is given by

$$F(x,\alpha,\beta,\lambda)=\left[\left(1+\frac{\beta}{x}\right)\exp\left(\frac{\beta e^{-\alpha x}}{x}\right)\right]^{-\lambda};x\geq 0,(\alpha,\beta,\lambda)>0 \tag{2.1}$$

The probability density function (PDF) corresponding to the newly proposed model is defined as follows:

$$\begin{aligned}f(x,\alpha,\beta,\lambda)=&\left(\frac{\beta\lambda}{x^2}\right)\exp\left(\frac{\beta e^{-\alpha x}}{x}\right)\left\{1+e^{-\alpha x}\left(1+\frac{\beta}{x}\right)(1+\alpha x)\right\}\\&\left[\left(1+\frac{\beta}{x}\right)\exp\left(\frac{\beta e^{-\alpha x}}{x}\right)\right]^{-(\lambda+1)};x\geq 0,(\alpha,\beta,\lambda)>0\end{aligned} \tag{2.2}$$

In the following section, we delve into different characteristics of the suggested model. These characteristics encompass the survival function, hazard rate function, reversed hazard rate function, cumulative hazard rate function, quantile function, and its behavior as it approaches its asymptotic limit.

2.1. Survival function

The survival function, denoted as S(x), represents the probability of enduring an event beyond a certain point x. It serves as the complement to the cumulative distribution function (CDF). Equation (2.3) provides the survival function for the proposed model.

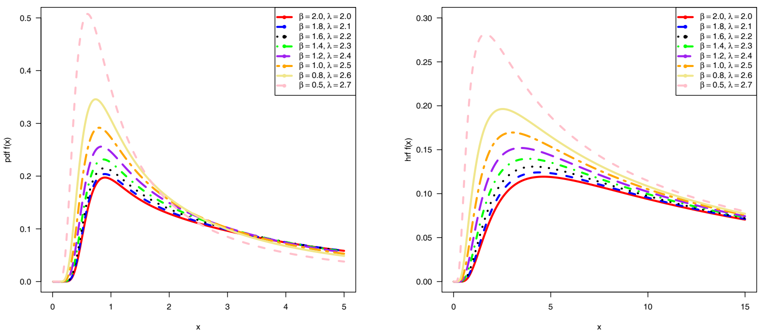
$$S(x)=1-\left[\left(1+\frac{\beta}{x}\right)\exp\left(\frac{\beta e^{-\alpha x}}{x}\right)\right]^{-\lambda};x\geq 0,(\alpha,\beta,\lambda)>0 \tag{2.3}$$

2.2. Hazard rate function

The hazard rate function, often symbolized as h(x), quantifies how quickly failures happen at a particular point in time. It's determined by dividing the probability density function (pdf) by the survival function S(x) of the distribution. In the proposed model, equation (2.4) offers a precise definition for h(x).

$$\begin{aligned}h(x)=&\left(\frac{\beta\lambda}{x^2}\right)\left\{1+e^{-\alpha x}\left(1+\frac{\beta}{x}\right)(1+\alpha x)\right\}\left[\left(1+\frac{\beta}{x}\right)\exp\left(\frac{\beta e^{-\alpha x}}{x}\right)\right]^{-(\lambda+1)}\\&\exp\left(\frac{\beta e^{-\alpha x}}{x}\right)\left\{1-\left\{\left(1+\frac{\beta}{x}\right)\exp\left(\frac{\beta e^{-\alpha x}}{x}\right)\right\}^{-\lambda}\right\}^{-1};x\geq 0,(\alpha,\beta,\lambda)>0\end{aligned} \tag{2.4}$$

Figure 1 presents two panels illustrating key aspects of the probability density curve and hazard rate curves for various parameter values. The left panel showcases the probability density curve, highlighting its variation as the parameters change. This variability signifies the model's adaptability to different types of datasets. Meanwhile, the right panel of Figure 1 displays hazard rate curves associated with specific parameter sets. These hazard rate curves exhibit patterns of both increasing and decreasing trends, as well as the distinctive inverted bathtub shape.



(a) Probability density (b) Hazard rate function

Figure 1. Probability density curve and hazard rate curve.

2.3. Reversed hazard rate function

The equation (2.5) defines the reversed hazard rate, denoted as hrex(x), for this model.

$$h_{rev}(x) = \left(\frac{\beta\lambda}{x^2}\right) \exp\left(\frac{\beta e^{-\alpha x}}{x}\right) \left\{1 + e^{-\alpha x}\left(1 + \frac{\beta}{x}\right)(1 + \alpha x)\right\} \left[\left(1 + \frac{\beta}{x}\right) \exp\left(\frac{\beta e^{-\alpha x}}{x}\right)\right]^{-1} \tag{2.5}$$

2.4. Cumulative hazard rate function

The equation (2.6) provides the cumulative hazard rate function, H(x), for the proposed model.

$$H(x) = -\ln S(x) = -\ln\left\{1 - \left[\left(1 + \frac{\beta}{x}\right) \exp\left(\frac{\beta e^{-\alpha x}}{x}\right)\right]^{-\lambda}\right\}; x \geq 0, (\alpha, \beta, \lambda) > 0 \tag{2.6}$$

2.5. Quantile function

The quantile function, an alternative to the cumulative distribution function (CDF), aids in the descriptive analysis of the model. It is defined for NTPD by equation (2.7).

$$\log\left(1 + \frac{\beta}{x}\right) + \left(\frac{\beta e^{-\alpha x}}{x}\right) + \frac{\log p}{\lambda} = 0; \quad 0 \leq p \leq 1 \tag{2.7}$$

2.6. Asymptotic behavior

We can examine the density function's behavior as it approaches zero and infinity by ensuring that $\lim_{x \to 0} f(x) = \lim_{x \to \infty} f(x)$. If the model follows these asymptotic properties, it will have a mode. This evaluation requires us to analyze the limits at both ends.

$$\lim_{x\to 0}\left(\frac{\beta\lambda}{x^2}\right)\left[\exp\left(\frac{\beta e^{-\alpha x}}{x}\right)\right]^{-\lambda}\left\{1+e^{-\alpha x}\left(1+\frac{\beta}{x}\right)(1+\alpha x)\right\}\left(1+\frac{\beta}{x}\right)^{-(\lambda+1)}=0$$
$$\lim_{x\to \infty}\left(\frac{\beta\lambda}{x^2}\right)\left[\exp\left(\frac{\beta e^{-\alpha x}}{x}\right)\right]^{-\lambda}\left\{1+e^{-\alpha x}\left(1+\frac{\beta}{x}\right)(1+\alpha x)\right\}\left(1+\frac{\beta}{x}\right)^{-(\lambda+1)}=0 \tag{2.8}$$

2.7. Skewness and Kurtosis

In this research, we employed Bowley's skewness coefficient, calculated using quantiles, as introduced by (Al-saiary et al., 2019), which is

$$\text{SK (B)} = \frac{\text{Q (3/4) +Q (1/4) -2*Q (1/2)}}{\text{Q(3/4)-Q(1/2)}}$$

The Octiles Kurtosis coefficient as presented in (Moors, 1988) is

$$\text{K}_\text{u}=\{\text{Q (0.375)-Q(0.625)-Q (0.125)+Q (0.875)}\}\{\text{Q (0.75)-Q(0.25)}\}^{-1}$$

3. Methods for Estimation of model constants

In the realm of literature, multiple techniques exist for estimating the parameters (constants) of the model. In this research, we employed three distinct approaches: maximum likelihood estimation, the least squares estimation method, and the Cramer-Von Mises estimation method.

3.1. Maximum Likelihood Estimation (MLE)

This estimation method depends on optimizing the model's log likelihood function. Imagine we have a random sample of 'n' items from MATE, which we'll represent as .In this situation; the log likelihood function can be formulated as follows:

$$\begin{aligned} l(\alpha,\lambda,\beta|\underline{x}) &= n\log(\lambda\beta) - 2\sum_{i=1}^{n}\log x_i + \sum_{i=1}^{n}\log\left[1+e^{-\alpha x_i}(1+\beta/x_i)(1+\alpha x_i)\right] \\ &+\beta\sum_{i=1}^{n}{x_i}^{-1}e^{-\alpha x_i} + \sum_{i=1}^{n}\log\left(1+\beta/x_i\right) + \sum_{i=1}^{n}\left(\beta {x_i}^{-1}e^{-\alpha x_i}\right)^{-(\lambda+1)} \end{aligned} \tag{3.1}$$

Once we have determined the derivatives of equation (3.1) with respect to α, β, and λ, we can move forward to calculate the first-order and second-order partial derivatives of the log-likelihood function. These derivatives are crucial for analyzing the behavior and properties of the likelihood function in our statistical or mathematical context. To estimate the parameters of the proposed model, we set the first-order derivatives to zero and solve for them. However, it's worth noting that solving these first-order partial derivatives analytically might not be practical, and we might have to resort to using computer programming to solve the nonlinear equations.

3.2. Estimation using Least-Square (LSE)

We begin with a series of arranged random variables, labeled as

$$\mathrm{X}_{(1)} < \mathrm{X}_{(2)} < \ \ldots < \mathrm{X}_{(\mathrm{n})}$$

. Next, we extract a random sample

$$\{X_1, X_2, \ \ldots, X_n\}$$

of size n from a distribution described by the function F(.). To establish a function A, we utilize the cumulative distribution function (CDF) of ordered statistics, represented as

$$F(X_{(i)})$$

,as outlined in equation (3.2).

$$A\left(x;\alpha,\lambda,\beta\right) = \sum_{i=1}^{n}\left[F(X_{(i)}) - \frac{i}{n+1}\right]^2 \tag{3.2}$$

To obtain the parameters of the proposed NTPD model, we can minimize function (3.2) with respect to the parameters and then solve for them.

3.3. Cramer-Von-Mises (CVM) method

We can estimate the parameters α,λ, and β by minimizing the function (3.3) through the utilization of this approach.

$$Z\left(X;\alpha,\lambda,\beta\right) = \frac{1}{12n} + \sum_{i=1}^{n}\left[F\left(x_{i:n}|\lambda,\beta\right) - \frac{2i-1}{2n}\right]^2 \tag{3.3}$$

To find both the first and second-order partial derivatives of function Z, we perform differentiation on equation (3.3) with respect to α,λ and β. Solving these nonlinear equations enables us to determine the estimated parameters.

4. Real Data Analysis

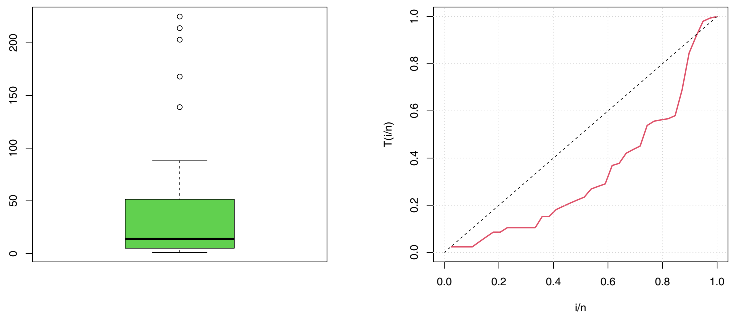
To evaluate the model's suitability, we utilized it with real data from the second wave of COVID-19 in Nepal. COVID-19, a global pandemic, hit Nepal, leading to severe cases of acute respiratory syndrome. This pandemic has seen multiple waves worldwide. In Nepal, the second wave of COVID-19 syndrome became prominently apparent in the first week of April,2021. Unfortunately, Nepal experienced a high mortality rate during this second wave. At the start of April, there was just one recorded death, but this number steadily rose. By May 14th, the daily death toll in Nepal had reached 203. As a result, this research aimed to forecast the fatalities occurring in Nepal during the second wave, spanning from April 1st to May 14th. The dataset, comprising a minimum of one daily reported death throughout this timeframe, was provided by

(Ministry of Health and Population of the Government of Nepal, 2021). 1, 1, 4, 2, 1, 1, 13, 5, 3, 5, 4, 5, 8, 8, 11, 10, 5, 5, 14, 28, 12, 18, 17, 35, 33, 19, 27, 37, 55, 58, 54, 50,53, 88, 139, 225, 168, 214, 203. Parameters are estimated using optim () function of R language programming (R Core Team, 2023). Figure 2 displays the boxplot in

Table 1. Summary statistics of the data.

Min.	Q1	Q2	Q3	Mean	Max.	S.D.	Skewness	Kurtosis
	1.00	5.00	14.00	51.50 42.03	225	61.954	1.9478	5.5907

the left panel and the TTT plot in the right panel.



(a) Boxplot. (b) TTT plot.

Figure 2. Boxplot and TTT plot of the data.

Parameters estimated using MLE is mentioned in table 2. Table 2 also contains the standard error of estimates of the parameters. Figure 3 exhibits both the histogram

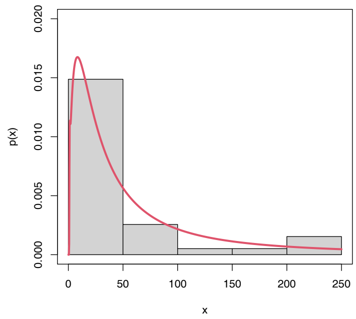
Table 2. Parameters estimated values using MLE and corresponding standard error (SE) of estimates

Parameters	MLE	SE
α	3.2857	0.7583
β	19.1460	11.9867
λ	0.8736	0.3192

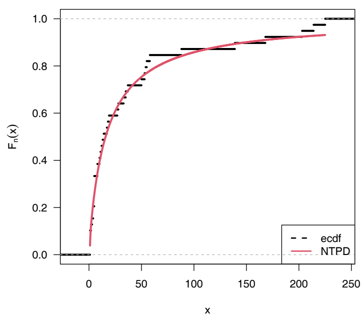
and the corresponding fitted density plot of the model. The right panel of Figure 3 showcases the comparison between the empirical cumulative distribution function (ecdf) and the fitted cdf of the proposed model.

To assess the reliability of the estimated parameter, two additional methods, namely the Least Square Estimation (LSE) and Cramer's von Mises Estimation (CVM), are employed. These methods help verify the consistency and accuracy of the estimated parameter, providing a robust evaluation of its reliability. Parameters estimated using these methods is very close.

Table 4 presents a comparison of estimation methods using four different information criteria values and negative log-likelihood values. The analysis reveals that the Maximum Likelihood Estimation (MLE) method exhibits the lowest information criteria values when compared to the other two methods. Consequently, we can confidently



(a) Histogram versus pdf



(b) Empirical versus fitted cdf

Figure 3. Histogram versus pdf and Empirical versus fitted cdf

Table 3. Estimated parameters using MLE, LSE and ,CVM.

Parameters	MLE	LSE	CVM
α	3.2857	4.4606	4.1056
β	19.1460	18.5512	15.9688
λ	0.8736	0.8701	0.9603

assert that the MLE method provides a superior fit to the COVID-19 second wave real dataset when compared to the alternative methods.

Table 4. Information criteria for different methods of estimation.

Methods	LL	AIC	BIC	CAIC	HQIC
MLE	-176.3767	358.7535	363.7442	359.4392	360.5441
CVM	-176.9073	359.8147	364.8054	360.5004	361.6053
LSE	-177.2643	360.5285	365.5192	361.2142	362.3191

Similarly, Table 5 represents the test statistics and corresponding p values for all methods of estimation

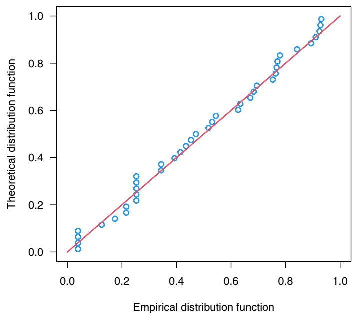
Table 5. Goodness of fit statistics and p values for different methods of estimation.

Methods	KS(p-value)	W(p-value)	A^2(p-value)
MLE	0.0807(0.9615)	0.3324 (0.9115)	0.0330(0.9672)
CVM	0.0809(0.9604)	0.2911(0.9444)	0.0293(0.9796)
LSE	0.0737(0.9840)	0.3150(0.9260)	0.0319(0.9743)

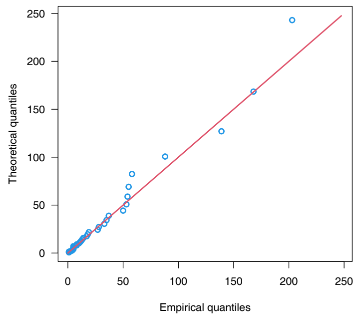
To assess the validity of the model, we have also generated P-P and Q-Q plots for the suggested model, which are displayed in Figure 4. These plots provide valuable insights into the suggested model's performance, helping us evaluate its accuracy and reliability. In Figure 4, we can observe the P-P plot and Q-Q plot, which offer a visual representation of how well the model aligns with the expected distribution.

4.1. Model Comparison

In this study, the proposed model is evaluated by comparing it with five other models that have been documented in existing literature. The five lifetime models under



(a) P-P plot



(b) Q-Q plot

Figure 4. P-P and Q-Q plot

consideration include the Exponentiated Marshall-Olkin Exponential (EMOE) distribution as presented by (Tharu et al., 2021), the Generalized Inverted Generalized Exponential (GIGE) distribution suggested by (Oguntunde et al., 2015), the Exponentiated Half Logistic Exponential (EHLE) distribution introduced by (Almarashi et al., 2018), the Exponentiated Generalized Inverted Exponential (EGIE) distribution created by (Oguntunde et al.,2014), and the Exponentiated Inverse Rayleigh (EIR) distribution presented by (Rao et al., 2019). In this comparative analysis, we seek to assess how the proposed model stacks up against these well-established models, considering various performance metrics and criteria. This evaluation will contribute valuable insights to the field, helping us better understand the strengths and weaknesses of each model in modeling lifetime data and informing potential applications in practical scenarios. Ultimately, the findings of this study will contribute to the ongoing dialogue surrounding the selection and application of lifetime models, offering valuable guidance to researchers, analysts, and professionals working in fields where the modeling of survival and failure times is crucial. Table 6 provides the estimated parameters for all these models, along with the standard error of estimates, using the given COVID- 19 second wave real dataset in Nepal.

Table 6. Estimated parameters of competing models.

Methods	α	β	λ	θ	γ	σ
NTPD	3.2858	19.1915	0.8737	-	-	-
EMOE	0.0073	0.0960	-	1.1669	-	-
EGIE	0.6767	24.7515	0.0582	-	-	-
GIGE	0.5785	-	1.6003	-	2.2031	-
EHLE	0.0102	0.4877	1.8705	-	-	-
EIR	0.2142	-	-	-	-	1.5483

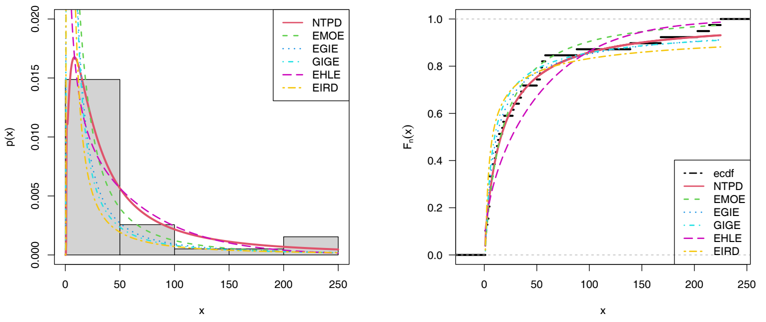
To compare the models, we have calculated various information criteria values for all of them and presented the results in Table 7. The proposed model exhibits the lowest information criteria values, suggesting that it is a better fit for the dataset compared to the other competing models. This superiority in information criteria values implies that the suggested model offers a more accurate representation of the data, outperforming the alternative models under consideration. These results underscore the robustness and effectiveness of the proposed model in capturing the underlying patterns and relationships within the dataset.

Figure 5 depicts two panels. The left panel shows a histogram compared to the

Table 7. Information criteria values for NTPD and competing models.

Methods	LL	AIC	BIC	CAIC
NTPD	-176.377	358.754	363.744	359.439
EMOE	-178.160	362.319	367.310	367.995
EGIE	-180.107	366.213	371.206	367.899
GIGE	-181.476	368.952	373.943	369.638
EHLE	-182.609	371.218	376.209	371.905
EIR	-184.401	372.802	376.129	373.135

fitted density function for all competing distributions, while the right panel displays the empirical cumulative distribution function (CDF) versus the fitted CDF for all the models.



(a) Histogram versus pdfs (b) Fitted cdf versus emperical cdfs

Figure 5. Histogram versus pdfs and fitted cdf versus emperical cdfs

We conducted a thorough comparison between our proposed model and both empirical and theoretical cumulative distributions. Our examination revealed a remarkable alignment between the empirical distribution and our model's theoretical cumulative distribution within the real dataset used for illustration. Furthermore, we assessed our model, denoted as NTPD, against the theoretical cumulative distributions of other models such as EMOE, EGIE, GIGE, and EIRD. Similarly, we compared the probability density function (PDF) of our model with those of competing models. The results of our analysis unequivocally demonstrate that our proposed model outperforms all other competitive models in fitting the given dataset, as illustrated in Figure 5. This superiority is evident in the way our model closely tracks the observed data points, thus providing a more accurate representation of the underlying distribution. The alignment of our model's theoretical cumulative distribution with the empirical data underscores its robustness and suitability for modeling the specific dataset under consideration.

5. Conclusion

In this research, we introduced a new distribution known as the Novel Three-Parameter Distribution (NTPD). Our primary goal was to utilize this distribution for modeling lifetime data, and our findings demonstrated its exceptional performance in the fields of reliability and survival analysis. The NTPD is characterized by a positively skewed and unimodal distribution. We conducted an in-depth examination of various statistical properties associated with the NTPD model. This investigation revealed that the model offers remarkable flexibility, accommodating both increasing and decreasing hazard functions, as well as an inverted bathtub-shaped hazard function. These insights were derived from a thorough graphical analysis of the Probability Density Function (PDF) and Hazard Rate Function (HRF) of the NTPD.

To estimate the model's parameters, we employed three distinct methods: Cramer's-von Mises Estimation (CVME), Least Squares Estimation (LSE), and Maximum Likelihood Estimation (MLE). These approaches provided valuable insights into the accuracy of our model's parameter estimation.

Furthermore, we put the NTPD distribution to the test by applying it to real-world COVID-19 second wave data from Nepal. The results of this application demonstrated the superior fitting performance of the NTPD distribution when compared to several other commonly used lifetime models. This underscores the potential of the NTPD as a valuable tool in the analysis of lifetime data, particularly in the context of complex and dynamic scenarios such as the COVID-19 pandemic.

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